No. of printed pages: 07

CLASS: XI
ANNUAL EXAMINATION (2024 – 25)
SUBJECT: MATHEMATICS (041)
SET-C1/C2 (Marking Key/Solutions)

Time: 3 Hrs. **Max. Mark: 80**

SECTION A
(Multiple Choice Questions) Each question carries 1 mark.

1/18. On the real axis, if $A = [0,3]$ and $B = [2,6]$, then $A \cap B$ is
a) $[2,3]$ b) $(2,3)$ c) $(0,6)$ d) $[0,6]$ **(a)**

2/17. Let $S = \{x: x \text{ is a positive multiple of } 3 \text{ less than } 100\}$, $P = \{x: x \text{ is a prime number less than } 20\}$. Then $n(S) + n(P) =$
a) 30 b) 33 c) 34 d) 41 **(d)**

3/16. If R be a relation from a set A to a set B then
a) $R = A \cup B$ b) $R = A \cap B$ c) $R \subseteq A \times B$ d) $R \subseteq B \times A$ **(c)**

4/15. Let $n(A) = 4$, then the number of relations on A is
a) 2^4 b) 2^8 c) $2^{4!}$ d) 2^{16} **(d)**

5/14. The value of $\sin 50° - \sin 70° + \sin 10°$ is equal to
a) 0 b) 1 c) 2 d) $\frac{1}{2}$ **(a)**

6/13. Which of the following numbers is/are rational:
a) $\sin 15°$ b) $\cos 15°$ c) $\sin 15° \cos 15°$ d) $\sin 15° \cos 75°$ **(c)**

7/12. If $i = \sqrt{-1}$ then $1 + i^2 + i^3 - i^6 + i^8$ is equal to
a) $2 - i$ b) 1 c) 3 d) -1 **(a)**

8/11. If $x, y \in R$, then $x + iy$ is a non-real complex number if
a) $x = 0$ b) $y = 0$ c) $x \neq 0$ d) $y \neq 0$ **(d)**

9/10. If $x^2 > 16$, then x belongs to
a) $(-4,4)$ b) $(0,4)$ c) $(4, \infty)$ d) $(-\infty, -4) \cup (4, \infty)$ **(d)**

10/9. In how many ways can be bowler take four wickets in a single 6 balls over?
a) 6 b) 15 c) 20 d) 30 **(b)**

11/8. If second term of a G.P. is 2 and the sum of its infinite term is 8, then its first term is
a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 2 d) 4 **(d)**

12/7. If the arithmetic and geometric means of two numbers are 10 and 8 respectively, then one number exceeds the other number by
a) 8 b) 10 c) 12 d) 16 **(c)**

13/6. The centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is (a, b) then $(2a + 3b)$ is
a) 0 b) 2 c) 3 d) 5 **(a)**

14/5. The locus of a point for which $y = 0, z = 0$ is
a) x −axis b) y −axis c) z −axis d) z and y axes **(a)**

15/4. The value of $\lim_{x \to 0} \frac{\log(1-3x)}{x}$ is
a) $-\frac{1}{3}$ b) -3 c) 3 d) $\frac{1}{3}$ **(b)**

16/3. If $xy = 4$, then $\frac{dy}{dx} =$
a) $-\frac{x}{y}$ b) $-\frac{4}{y^2}$ c) $4x^2$ d) $-\frac{4}{x^2}$ **(d)**

17/2. The value of $\lim_{x\to\pi}\frac{\sin x}{x-\pi}$ is

a) 1 b) 2 c) -1 d) π **(c)**

18/1. Three unbiased coins are tossed. If the probability of getting at least 2 tails is p, then value of 8p is

a) 0 b) 1 c) 3 d) 4 **(d)**

ASSERTION- REASONING

Of the following statements, choose the correct one.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true and R is not correct explanation of A.
c) A is true but R is false. d) A is false but R is true.

19/20. Assertion (A): Product of five consecutive natural numbers is divisible by 4!.
Reason (R): Product of n consecutive natural number is divisible by $(n+1)!$ **(c)**

20/19. Assertion (A): The ratio of the radii of two circles at the centres of which two equal arcs subtend angles of $30°$ and $70°$ is 21:10.
Reason (R): Number of radians in an angle subtended at the centre of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle. **(d)**

SECTION B

(All Questions are compulsory. In case of internal Choice, attempt any one question only)

21/25. Given $A=\{1,2,3,5\}$ and $B=\{4,6,9\}$. Define a relation R from A to B by $R=\{(x,y):|x-y|=odd, x\in A, y\in B\}$. Write R in roster form. Also, find its domain and range.

Sol. $R=\{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$. Domain: A, Range: B **(1+$\frac{1}{2}\times$ 2) M**

OR

Draw the graph of the Constant function $f:R\to R; f(x)=2\ \forall x\in R$. Also, find its domain and range.

Sol. Graph (**1M**), Domain: R and Range: $\{2\}$ **(0.5+0.5) M**

22/24. Show that $\tan 3x\tan 2x\tan x=\tan 3x-\tan 2x-\tan x$.

Sol. $\tan 3x=\frac{\tan 2x+\tan x}{1-\tan 2x\tan x}\implies \tan 3x(1-\tan 2x\tan x)=\tan 2x+\tan x$

$\implies \tan 3x\tan 2x\tan x=\tan 3x-\tan 2x-\tan x$. **(1+1) M**

23/23. If the extremities of a diagonal of a square are $(1,-2,3)$ and $(2,-3,5)$ then find the length of the side of square.

Sol. Length of diagonal= $\sqrt{1^2+(-1)^2+2^2}=\sqrt{6}$. Let side of the square be a.

$\implies a^2+a^2=\left(\sqrt{6}\right)^2 \implies 2a^2=6 \implies a^2=3 \implies a=\sqrt{3}$. **(1+1) M**

24/22. Evaluate: $\lim_{x\to 0}\left(\frac{\sin 2x+3x}{2x+\tan 7x}\right)$.

Sol. $\lim_{x\to 0}\left(\frac{\sin 2x+3x}{2x+\tan 7x}\right)=\lim_{x\to 0}\frac{\frac{\sin 2x}{x}+3}{2+\frac{\tan 7x}{x}}=\frac{2+3}{2+7}=\frac{5}{9}$. **(1+1) M**

25/21. If A and B are mutually exclusive events such that $P(A)=0.35$ and $P(B)=0.45$, then find $P(A'\cap B')$.

Sol. $P(A'\cap B')=P(A\cup B)'=1-P(A\cap B)=1-[P(A)+P(B)]=1-0.80=0.20$

OR

If A, B and C are three mutually exclusive and exhaustive events of an experiment such that $3P(A) = 2P(B) = P(C)$, then find the value of P(A).

Let $3P(A) = 2P(B) = P(C) = k \implies P(A) = \frac{k}{3}, P(B) = \frac{k}{2}, P(C) = k$

Now, $\frac{k}{3} + \frac{k}{2} + k = 1 \implies 2k + 3k + 6k = 6 \implies k = \frac{6}{11}$. $P(A) = \frac{2}{11}$. **(1+1) M**

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26/31. If $A = \{1,3,5,7,11,13,15,17\}$, $B = \{2,4,6,8, \dots, 18\}$ and $U = \{1,2,3,4, \dots, 20\}$ where U is universal set then find $A' \cup [(A \cup B) \cap B']$.

Sol. $A' = \{2,4,6,8,9,10,12,14,16,18,19,20\}, B' = \{1,3,5,7,9,11,13,15,17,19,20\}$,
$A \cup B = \{1,2,3,4,5,6,7,8,10,11,12,13,14,15,16,17,18\}$,
$[(A \cup B) \cap B'] = \{1,3,5,7,11,13,15,17\}$ and $A' \cup [(A \cup B) \cap B'] = U$. **(1+1+1) M**

27/30. Evaluate: $(1+i)^6 + (1-i)^3$ and find its conjugate.

Sol. $(1+i)^6 + (1-i)^3 = (1+i^2+2i)^3 + (1-i^3-3i+3i^2) = 8i^3 + 1 + i - 3i - 3$
$= -2 - 10i$. Conjugate is $-2 + 10i$. **(2+1) M**

28/29. Solve for x: $|x+1| + |x| > 3$.

Sol. Case I: $x \in (-\infty, -1)$ then, $|x+1| + |x| > 3 \implies -x - 1 - x > 3 \implies x < -2$.
Case II: $x \in [-1,0)$ then, $|x+1| + |x| > 3 \implies x + 1 - x > 3 \implies 1 > 3$ not possible.
Case III: $x \in [0, \infty)$ then, $|x+1| + |x| > 3 \implies x + 1 + x > 3 \implies x > 1$
Combining all cases, result is $x \in (-\infty, -2) \cup (1, \infty)$. **(2+1) M**

OR

A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% a acid solution must be added to it so that acid content in the resulting mixture will be

more than 15% but less than 18%?

Sol. Let x litres of 30% acid solution is required to be added.

Then total mixture= $(x + 600)\ l$. Therefore, $30\%\ x + 12\%\ of\ 600 > 15\%\ of\ (x + 600)$

And $30\%\ x + 12\%\ of\ 600 < 18\%\ of\ (x + 600)$

Also, $\frac{15}{100}(x + 600) < \frac{30}{100}x + \frac{7200}{100} < \frac{18}{100}(x + 600)$

$\implies 15x + 9000 < 30x + 7200 < 18x + 10800 \implies 120l < x < 300l$. **(1+1+1) M**

29/28. A student has to answer 10 questions, choosing at least 4 from each of parts A and B. If there are 6 questions in Part A and 7 in Part B, in how many ways can the student choose 10 questions?

Sol. The required number of ways is $C(6,4) \times C(7,6) + C(6,5) \times C(7,5) + C(6,6) \times C(7,4)$
$= 105 + 126 + 35 = 266$. **(2+1) M**

OR

Find the number of words with or without meaning which can be made using all the letters of the word $AGAIN$. If these words are written as in a dictionary, what will be the 49th word?

Sol. number of words using all the letters of the word $AGAIN = \frac{5!}{2!} = 60$

Number of words beginning with $A = 4! = 24$, Number of words beginning with $G = \frac{4!}{2!} = 12$

Number of words beginning with $I = \frac{4!}{2!} = 12$,

Number of words so far $= 24 + 12 + 12 = 48$ and 49th word is $NAAGI$. **(2+1) M**

30/-. Find $(a+b)^4 - (a-b)^4$. Hence, evaluate $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$.

Sol. $(a+b)^4 - (a-b)^4$

$= (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) - (a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4)$

$= 8(a^3b + ab^3)$. Hence, $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 40\sqrt{6}$. **(2+1) M**

-/27. Find $(x+1)^6 + (x-1)^6$. Hence, evaluate $\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6$.

Sol.

$(x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1) + (x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1)$

$= 2(x^6 + 15x^4 + 15x^2 + 1)$

Now, $\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6 = 2(8 + 15 \times 4 + 15 \times 2 + 1) = 2 \times 99 = 198$. **(2+1) M**

31/26. Find $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} 5x-4, & 0 < x \le 1 \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$.

Sol. $LHL = 1 = RHL \rightarrow \lim_{x\to 1} f(x)$ exists and equal to 1. **(1+1+1) M**

OR

Differentiate: $f(x) = \frac{\sec x - 1}{\sec x + 1} + x^2 \sin x \ w.r.t.x.$

Sol. $\frac{d}{dx}\left(\frac{\sec x-1}{\sec x+1}\right) = \frac{(\sec x+1)(\sec x-1)' - (\sec x-1)(\sec x+1)'}{(\sec x+1)^2} = \frac{2\sec x \tan x}{(\sec x+1)^2}$ and **(1 M)**

$\frac{d}{dx}(x^2 \sin x) = x^2 \cos x + 2x \sin x$. Hence, $f'(x) = \frac{2\sec x \tan x}{(\sec x+1)^2} + x^2 \cos x + 2x \sin x$.

(1+1) M

SECTION D

(This section comprises of long answer type questions (LA) of 5 marks each.)

32/35. Find the value of the expression:

$3\left[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right]$

Sol. $3[\cos^4\alpha + \sin^4\alpha] - 2[\cos^6\alpha + \sin^6\alpha]$

$= 3[\cos^4\alpha + \sin^4\alpha] - 2[(\cos^2\alpha + \sin^2\alpha)(\cos^4\alpha + \sin^4\alpha - \cos^2\alpha \sin^2\alpha)]$

$= 3[\cos^4\alpha + \sin^4\alpha] - 2[(\cos^4\alpha + \sin^4\alpha - \cos^2\alpha \sin^2\alpha)]$

$= \cos^4\alpha + \sin^4\alpha + 2\cos^2\alpha \sin^2\alpha = (\cos^2\alpha + \sin^2\alpha)^2 = 1$. $(\mathbf{5 \times 1})$ **M**

OR

If α and β are the solutions if the equation $a\tan\theta + b\sec\theta = c$, then show that $\tan(\alpha+\beta) = \frac{2ac}{a^2-c^2}$.

Sol. $a\tan\theta + b\sec\theta = c \rightarrow (a\tan\theta - c)^2 = (b\sec\theta)^2 = b^2(1+\tan^2\theta)$

$(a^2-b^2)\tan^2\theta - 2ac\tan\theta + c^2 - b^2 = 0,$

Since α and β are the roots, $\tan\alpha + \tan\beta = \frac{2ac}{a^2-b^2}$, $\tan\alpha\,.\tan\beta = \frac{c^2-b^2}{a^2-b^2}$.

$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta} = \frac{2ac}{a^2-c^2}$. **(2+1+2) M**

33/34. If n Arithmetic Means are inserted between 1 and 31 such that the ratio of the 7th and $(n-1)^{th}$ means is 5:9. Find n.

Sol. 1, $A_1, A_2, \ldots, \ldots, A_n, 31$ are in A.P. $d = \frac{30}{n+1}, \frac{A_7}{A_{n-1}} = \frac{a+7d}{a+(n-1)d} = \frac{5}{9}$.

$\Rightarrow 4a = 5(n-1)d - 63d \Rightarrow n = 14.$

34/-. A line is such that its segment between the lines $5x - y + 4 = 0\ and\ 3x + 4y - 4 = 0$ is bisected at the point (1,5). Obtain its equation.

Sol. Given lines are $5x - y + 4 = 0\ and\ \ 3x + 4y - 4 = 0$

Let the required line intersects given lines at the points $(\alpha, \beta)\ and\ (a, b)$

$\Rightarrow 5\alpha - \beta + 4 = 0\ and\ 3a + 4b = 4 \quad \Rightarrow \beta = 5\alpha + 4\ and\ b = \frac{4-3a}{4}$

Mid-point of $(\alpha, \beta) and\ (a, b)$ is (1,5). Hence

$$\frac{\alpha + a}{2} = 1\ and\ \frac{\beta + b}{2} = 5\ \Rightarrow\ \alpha + a = 2\ and\ 5\alpha + 4 + \frac{4-3a}{4} = 10$$

On solving, $\alpha = \frac{26}{23}$, $a = \frac{20}{23}$ $and\ \beta = \frac{222}{23}$

Equation of line passing through (1,5) and $(\frac{26}{23}, \frac{222}{23})$ is $107x - 3y - 92 = 0$.$(\mathbf{5 \times 1})$ **M**

-/33. If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at (1,2), then find the equation of sides of the square passing through this vertex.

Sol. Let ABCD be the given square and the coordinates of the vertex D be (1,2). Slope of the diagonal BD is $\frac{8}{15}$. Angles made by BD with sides AD and DC is 45°. Let slope of DC is m.

$\tan 45° = \frac{m-\frac{8}{15}}{1+\frac{8m}{15}}$ which gives $m = \frac{23}{7}$. Side AD is perpendicular to DC ∴ slope od AD= $-\frac{7}{23}$.

Equation of DC: $y - 2 = \frac{23}{7}(x-1) \Rightarrow 23x - 7y - 9 = 0.$

Equation of AD: $y - 2 = -\frac{7}{23}(x-1) \Rightarrow 7x + 23y - 53 = 0.$ **(2+1+2) M**

35/32. Calculate the mean, variance and standard deviation for the following distribution:

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Sol.

CLASS	Frequency	Mid-point (x_i)	$f_i x_i$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135

70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178
	50		3100		10050

Mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3100}{50} = 62$ and Variance $\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} = \frac{10050}{50} = 201$

Standard Deviation $(\sigma) = \sqrt{201} = 14.18$. **(2 M for table +1+1+1) M**

OR

The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6, find the other two observations.

Sol. Let x , y be the other two observations.

$\Rightarrow 4.4 = \frac{1+2+6+x+y}{5}$ $\Rightarrow 22 = 9 + x + y$ $\Rightarrow x + y = 13$

Also, 8.24$= \frac{1}{5}[(3.4)^2 + (2.4)^2 + (1.6)^2 + x^2 + y^2 - 2 \times 4.4 \times (x + y) + 2 \times (4.4)^2]$

$\Rightarrow x^2 + y^2 = 97$ and $x^2 + y^2 + 2xy = 169$, $2xy = 72$

$x - y = \pm 5, x = 9, y = 4.$

Thus, the remaining observations are 4 and 9. **(1+1+2+1) M**

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

36/37. 5 students Arpit, Akshay, David, Aditya and Raghav, each of age 16 years are selected from Class 11 for the school football team. Their heights in centimetres are 182,181,180.5,180 and 179.5 respectively.

Answer the following questions by taking A as the set of ages of students and B as set of heights of students.

i) Write sets A and B in roster form.

ii) Find the cartesian product of two sets A and B.

iii)Find the total number of relations from A to B.

OR

Check whether $A \times B$ and $B \times A$ are functions or not.

Sol. i) Since all the students have age 16 years.

$\therefore A = \{16\}$

And $B = \{182,181,180.5,180,179.5\}$ **(1 M)**

ii) $A \times B = \{(16,182), (16,181), (16,180.5), (16,180), (16,179.5)\}$ **(1 M)**

iii)Number of relations$= 2^{n(A) \times n(B)} = 2^{1 \times 5} = 2^5 = 32.$

OR

$A \times B = \{(16,182), (16,181), (16,180.5), (16,180), (16,179.5)\}$. Here the same element 16 corresponds to 5 different images. So, it is not a function.

But $B \times A = \{(182,16), (181,16), (180.5,16), (180,16), (179.5,16)\}$. Here every element

of set B has a unique image. Therefore, this relation from B to A is a function. **(2 M)**

37/36. Window is in the shape of parabola with an isosceles triangle inscribed in it. The triangle is formed in such a way that the vertices of triangle coincide with vertex of parabola and end points of the latus rectum. The equation of parabola is given by $x^2 = 24y$.

Based on the above information, answer the following:

i) Find the vertices of triangle.

ii) Find the length of the altitude of the triangle.

iii) Find the coordinates of the focus of the parabola and the length of the latus rectum.

OR

Find the area of the triangle.

Sol. i) Vertices of triangle are: $(0,0), (12,6), (-12,6)$. **(1 M)**

ii) Length of the altitude: 6 units **(1 M)**

iii)Focus: $(0,6)$ and Length of latus rectum: 24 units **(2 M)**

OR

Area of the triangle: $\frac{1}{2} \times 24 \times 6 = 72$ sq. units. **(2 M)**

38/38. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that

i) It will get at least one of the 2 awards.

ii) It will get only one of the awards.

Sol. A: an award for its design and B: an award for the efficient use of materials

$P(A) = 0.48, P(B) = 0.36, P(A \cap B) = 0.2$

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.48 + 0.36 - 0.2 = 0.64$. **(2 M)**

ii) Required probability: $P(A \cap M') \cup P(A' \cap B) = P(A \cup B) - P(A \cap B)$

$= 0.64 - 0.2 = 0.44$ **(2 M)**