No. Of Printed Pages: 12

CLASS XII
POST MID-TERM EXAM (2024 – 25)
MATHEMATICS (041)
Set A1/A2
MARKING KEY

Time :3hrs. **M.M.80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 case-based assessment (4 marks) with sub parts.
7. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

SECTION A

(1 mark for each correct answer)

Q1/18) X and Y are two matrices such that the transpose of (X + Y) is $\begin{bmatrix}2 & -1\\7 & 6\end{bmatrix}$.

$If\ Y = \begin{bmatrix}3 & -2\\6 & 0\end{bmatrix}$, then which of the following is correct?

(a) X = $\begin{bmatrix}-1 & 1\\1 & 6\end{bmatrix}$ (b) X = $\begin{bmatrix}-1 & 9\\-7 & 6\end{bmatrix}$ (c) X = $\begin{bmatrix}1 & -9\\7 & -6\end{bmatrix}$ (d) X = $\begin{bmatrix}-4 & 4\\0 & 7\end{bmatrix}$ **(b)**

Q2/17) If $\begin{bmatrix}-5 & 6 & 7\\6 & 2 & m^2-6\\7 & 3 & 0\end{bmatrix}$ is a symmetric matrix, then $m =$

(a) 0 (b) -2 (c) 2 (d) ± 3 **(d)**

Q3/16) If for a square matrix B, $B.(adjB) = \begin{bmatrix}-25 & 0 & 0\\0 & -25 & 0\\0 & 0 & -25\end{bmatrix}, then\ |B| + |adj\ B| =$

(a)$25^2 \times 24$ (b) (-25) + 1 (c) 25×24 (d) $(-25)^2 + 25$ **(c)**

Q4/15) For the matrix $A = \begin{bmatrix}1 & -1 & 2\\3 & 0 & -2\\1 & 0 & 3\end{bmatrix}$, the value of $|A^{-1}|$ will be

$(a)\ \frac{1}{11}$ (b)$-\frac{1}{11}$ (c) -11 (d) 11 **(a)**

Q5/14) The domain of the function $cos^{-1}(2x-3)\ is$

(a) $[-1,1]$ (b) $[0,\pi]$ (c) $[1,2]$ (d) $(-1,1)$ **(c)**

Q6/13) The rate of change of the surface area of the sphere of radius r when the radius is increasing at the rate of 2 cm/s is proportional to

(a)$\frac{1}{r^2}$ (b) $\frac{1}{r}$ (c) r (d) r^2 **(c)**

Q7/12) If for a square matrix A, $A^2 - 3A + I = 0$ & $A^{-1} = xA + y\,I, then\ \ value\ of\ x + y\ is$

(a) -2 (b) 2 (c) 3 (d) -3 **(b)**

Q8/11) $\int \frac{dx}{(1-x)\sqrt{x}} =$

(a) $\frac{1}{2} log \left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right| + c$ (b) $-\frac{1}{2} log \left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right| + c$ (c) $log \left|\frac{1-\sqrt{x}}{1+\sqrt{x}}\right| + c$ (d) $log \left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right| + c$ **(d)**

Q9/10) If $\int e^{-\log x} dx = f(x) + k, then\ f(x)$ is

(a) $\frac{x^3}{3}$ (b) $log|x|$ (c) $-\frac{1}{x}$ (d) $\frac{1}{x}$ **(b)**

Q10/9) Area bounded by the parabola $y^2 = 4ax$ and its latus-rectum, is

(a) $8a^2$ (b) $\frac{4a^2}{3}$ (c) $\frac{8a^2}{3}$ (d) $\frac{3a^2}{4}$ **(c)**

Q11/8) The corner points of the feasible region determined by the system of linear constraints are (2, 4), (6, 7), (0, 8). Let Z = 3x – 5y be the objective function. The difference between maximum and minimum value of Z is

(a)26 (b) 14 (c) 17 (d) 23 **(a)**

Q12/7) The solution of $\frac{dy}{dx} - \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is

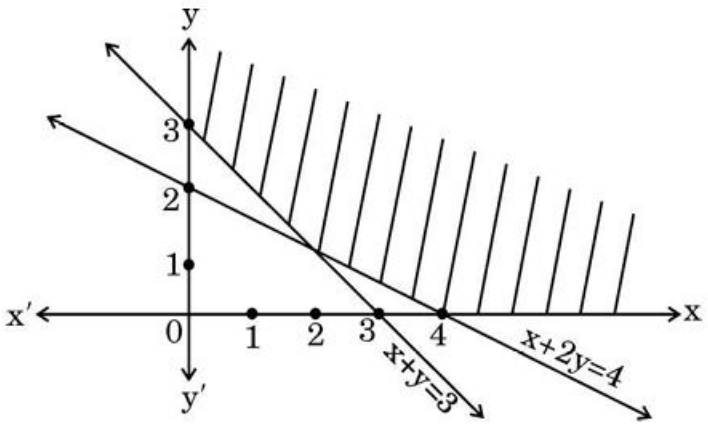
(a) $sin^{-1} x + y = c$ (b) $sin^{-1} x - sin^{-1} y = c$

(c) $sin^{-1} x + \sin^{-1} y = c$ (d) $tan^{-1} x + cot^{-1} y = c$ **(b)**

Q13/6) If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ is a non-zero scalar, then $\lambda\vec{a}$ is unit vector if

(a) $\lambda = 1$ (b) $\lambda = -1$ (c) $a = |\lambda|$ (d) $a = \frac{1}{|\lambda|}$ **(d)**

Q14/5) The feasible region of a linear programming problem is shown in the figure below:



Which of the following are the possible constraints?

(a) $x + 2y \geq 4, x + y \leq 3, x, y \geq 0$ (b) $x + 2y \leq 4, x + y \leq 3, x, y \geq 0$
(c) $x + 2y \geq 4, x + y \geq 3, x, y \geq 0$ (d) $x + 2y \geq 4, x + y \leq 3, x, y \leq 0$ **(c)**

Q15/4) If m and n are the order and degree, respectively of the differential equation

$7x\left(\frac{dy}{dx}\right)^4 - \frac{d^3y}{dx^3} - 6y = log\, x$, then the value of m + n is

(a) 1 (b) 2 (c) 3 (d) 4 **(d)**

Q16/3) If $(\hat{\imath} + 3\hat{\jmath} + 8\hat{k}) \times (3\hat{\imath} - \lambda\hat{\jmath} + \mu\hat{k}) = \vec{0}$, then the value of λ is

(a) 27 (b) 9 (c) –9 (d) – 1 **(c)**

Q17/2) Value of the constant 'k' so that the function $f(x) = \begin{cases} \frac{k|x|}{x}, if x < 0 \\ 3, if x \geq 0 \end{cases}$ is continuous at $x = 0$, is

(a) -3 (b) 3 (c) $-\frac{1}{\sqrt{2}}$ (d) $-\frac{\pi}{4}$ **(a)**

Q18/1) If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is equal to

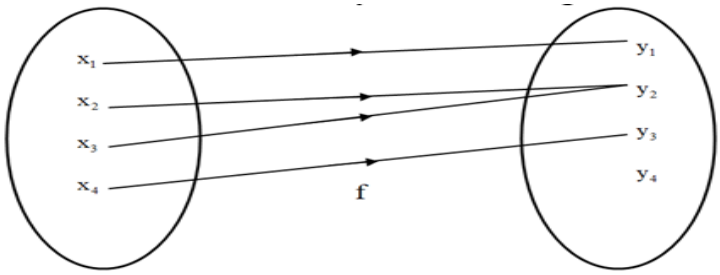
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) 1 **(c)**

ASSERTION-REASON BASED QUESTION:

In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19/20) **Assertion(A):** A function f shown below by the arrow diagram, is one-one.



Reason (R): A function $f: A \to B$ is one-one if $f(\alpha) = f(\beta)$ implies $\alpha = \beta$ for all $\alpha, \beta \in A$
Ans) **(d)**

Q20/19) **Assertion(A):** The function $f(x) = \begin{cases} 5 - 2x^2, & if\ x > 1 \\ x^2 + 1, & if\ x \leq 1 \end{cases}$ is not differentiable at x = 1.

Reason (R): A real function continuous at x = c is always differentiable as well at x = c.
Ans) **(c)**

SECTION B

Q21) SET A1) Consider the statement "There exists at least one value of $b \in \boldsymbol{R}$ *for which* $f(x) = \frac{b}{x}, b \neq 0$ *is strictly increasing in* $\boldsymbol{R} - \{0\}$. State True or False. Justify your answer.
/ Q25) SET A2) Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing for all $x \in \boldsymbol{R}$.
Ans) SET A1) For strictly increasing function, we have $\frac{df(x)}{dx} > 0$ *or* $-\frac{b}{x^2} > 0 \implies b < 0$ *and* $x \neq 0$.

The given statement is true for all $b < 0$ *and* $x \neq 0$ (1+1)

/SET A2) $f'(x) = e^x + e^{-x} + 1 - \frac{1}{1+x^2} = e^x + \frac{1}{e^x} + \frac{x^2}{1+x^2} > 0$ *for all* $x \in \boldsymbol{R}$.

Thus, f(x) is strictly increasing on **R.** (1 + 1)

Q22/24) If the projection of the vector $\lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on the vector $2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4, then find the value of λ.
Ans)

We know that projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} \Rightarrow 4 = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$ ...(*i*)

Now, $\vec{a}.\vec{b} = 2\lambda + 6 + 12 = 2\lambda + 18$ also $|\vec{b}| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{4 + 36 + 9} = 7$

Putting in (*i*), we get

$$4 = \frac{2\lambda + 18}{7} \Rightarrow 2\lambda = 28 - 18 \Rightarrow \lambda = \frac{10}{2} = 5$$

(1+1)

Q23/23) Find $\frac{dx}{dy}$ if $x = a\cos^2 t$ *and* $y = b\sin^3 t$

OR

If $y = \sqrt{\cos x + y}$, then prove that $\frac{dy}{dx} = \frac{\sin x}{1-2y}$

Ans) $x = a\cos^2 t$ *and* $y = b\sin^3 t \therefore \frac{dx}{dt} = a(-2\cos t.\sin t)$ and $\frac{dy}{dt} = b.3\sin^2 t.\cos t$

Therefore $\frac{dx}{dy} = \frac{dx/dt}{dy/dt} = -\frac{2a}{3b}\operatorname{cosec} t$ (1+1)

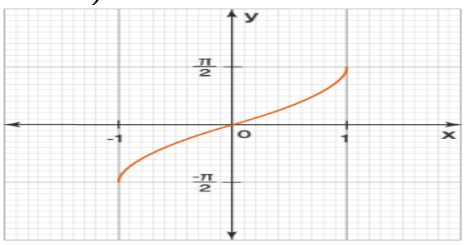
OR

$y = \sqrt{\cos x + y} \implies y^2 = \cos x + y \implies 2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx} \implies \frac{dy}{dx} = \frac{\sin x}{1-2y}$ (1+1)

Q24) SET A1) Find the value of k if $\sin^{-1}\left(k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right) = \frac{\pi}{3}$.
/ Q22) SET A2) Draw the graph of $y = \sin^{-1} x$, *where* $x \in [-1, 1]$. *Also, find its range*
Ans) SET A1)

$\sin^{-1}\left(k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right) = \frac{\pi}{3} \implies k\tan\left(2 \times \frac{\pi}{6}\right) = \sin\frac{\pi}{3} \implies k\tan\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ *or* $k = \frac{1}{2}$ (1+1)

/SET A2)



(Graph 1) Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (1)

Q25/21) Find a vector $\vec{r}$ of magnitude $5\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ *and* $\frac{\pi}{2}$ with the y-axis and z-axis, respectively and an acute angle α *with X axis*.

OR

A person standing at O (0, 0, 0) is watching an aeroplane which is at the coordinate point A (4, 0, 3). At the same time, he saw a bird at the coordinate point B (0, 0, 1). Find the angles which $\overrightarrow{BA}$ makes with X, Y and Z axes.

Ans) Let the vector make an acute angle α *with positive X axis, given that* $\beta = \frac{\pi}{4}$ *and* $\gamma = \frac{\pi}{2}$

$Now, cos^2\alpha + cos^2\beta + cos^2\gamma = 1$ *or* $cos^2\alpha = 1 - \left(\frac{1}{2} + 0\right) = \frac{1}{2}$ *or* $\alpha = \frac{\pi}{4}$

Thus, $\hat{r} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$ *or* $\vec{r} = 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right) = 5\hat{\imath} + 5\hat{\jmath}$ (1+1)

OR

$\overrightarrow{BA} = 4\hat{\imath} + 2\hat{k}$

$\widehat{BA} = \frac{4}{2\sqrt{5}}\hat{\imath} + \frac{2}{2\sqrt{5}}\hat{k} = \frac{2}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{k}$

So, the angles made by the vector $\overrightarrow{BA}$ with the x, y and the z axes are respectively

$cos^{-1}\left(\frac{2}{\sqrt{5}}\right), \frac{\pi}{2}, cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$.

(1+1)

SECTION C

Q26/31) Evaluate: $\int_0^{\pi/2} [\log sinx - \log(2\cos x)]\, dx$

OR

Evaluate $\int \frac{(3sinx-2)cosx}{(4+\, sin^2x-4sinx)} dx$

Ans) $I = \int_0^{\pi/2} [\log sinx - \log(2\cos x)]\, dx = \int_0^{\pi/2} log\left(\frac{\sin x}{2\cos x}\right) dx = \int_0^{\pi/2} log\left(\frac{\tan x}{2}\right) dx$

$\Rightarrow I = \int_0^{\pi/2} log\left(\frac{\cot x}{2}\right) dx$ *or* $2I = \int_0^{\pi/2} log\left(\frac{\tan x}{2} \times \frac{\cot x}{2}\right) dx = \int_0^{\pi/2} log\left(\frac{1}{4}\right) dx$

$\Rightarrow 2I = \frac{\pi}{2} log\left(\frac{1}{4}\right)$ *or* $I = -\frac{\pi}{2}\log 2$ (1.5 + 1.5)

OR

$Let\ I = \int \frac{(3sinx-2)cosx}{(4+\, sin^2x-4sinx)} dx = \int \frac{(3t-2)dt}{(4+t^2-4t)} = \int \frac{(3t-2)dt}{(4+t^2-4t)} = \int \frac{(3t-2)dt}{(t-2)^2}$ *where* $\sin x = t$

Now, $\frac{(3t-2)}{(t-2)^2} = \frac{A}{(t-2)} + \frac{B}{(t-2)^2}, comparing\ the\ coefficients, we\ get, A = 3\ and\ B = 4$

Thus, $I = \int \frac{3dt}{(t-2)} + \int \frac{4dt}{(t-2)^2} = 3\ log|t-2| + 4\left(\frac{-1}{(t-2)}\right) + c = 3\ log|sinx - 2| + 4\left(\frac{1}{(2-sinx)}\right) + c$

(1+1+1)

Q27/30) The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6, 0 \le x \le 12$, "m" being a constant, where f(x) is the temperature in ^{0}F at x days. If 6 is a critical point of the function, then find the value of the constant "m". Also, find the points of absolute maximum and absolute minimum in the interval [0, 12] along with the absolute maximum and minimum values.

Ans) At the critical point, $f'(x) = 0$ *or* $-0.2\,x + m = 0, as\ x = 6\ is\ a\ critical\ value$

So, $-0.2 \times 6 + m = 0$ *or* $m = 1.2, thus\ f'(x) = -0.2\,(x - 6)$

Now, f (6) = 102.2 ^{0}F, f (0) = 98.6 ^{0}F and f (12) = 98.6 ^{0}F

Thus, x = 6 is a point of absolute maxima and abs max value = 102.2^0F and x= 0 and 12 are points of absolute minima and 98.6 ^{0}F is the absolute min value. (1+1+1)

Q28/29) Find the vector equation of the line passing through the point (1, 3, – 7) and perpendicular to the two lines

$$\frac{x-6}{3}=\frac{y+23}{-16}=\frac{z-0}{7} \text{ and } \frac{x-5}{3}=\frac{y+9}{8}=\frac{z+5}{-5}$$

OR

Using vectors, derive the expression for evaluating the area of a parallelogram in terms of its diagonal vectors. Hence, find the area of a parallelogram whose diagonals are given by $3\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $\hat{\imath}-2\hat{\jmath}+4\hat{k}$.

Ans) *Required equation of the line is* $\vec{r}=\vec{a}+\lambda\vec{b}$ *where* $\vec{a}=\hat{\imath}+3\hat{\jmath}-7\hat{k}$

and $\vec{b}$ *is in the direction of the cross product of the direction vectors of the given lines.*

So, $\vec{b}$ *is parallel to* $\vec{b}_1\times\vec{b_2}$ *where* $\vec{b}_1=3\hat{\imath}-16\hat{\jmath}+7\hat{k}$ *and* $\vec{b_2}=3\hat{\imath}+8\hat{\jmath}-5\hat{k}$

$$Now,\ \vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{\imath} & \hat{\imath} & \hat{k}\\ 3 & -16 & 7\\ 1 & 3 & -7\end{vmatrix}=24\hat{\imath}+36\hat{\jmath}+72\,\hat{k}$$

$$Thus,\ the\ required\ equation\ of\ the\ line\ is\ \vec{r}=\vec{a}+\lambda\vec{b}=\left(\hat{\imath}+3\hat{\jmath}-7\hat{k}\right)+\lambda\left(24\hat{\imath}+36\hat{\jmath}+72\hat{k}\right)$$

$$or\ \vec{r}=\left(\hat{\imath}+3\hat{\jmath}-7\hat{k}\right)+12\lambda\left(2\hat{\imath}+3\hat{\jmath}+6\hat{k}\right)=\left(\hat{\imath}+3\hat{\jmath}-7\hat{k}\right)+\mu\left(2\hat{\imath}+3\hat{\jmath}+6\hat{k}\right)$$ (2 + 1)

OR

Let $\vec{a}$ and $\vec{b}$ *represent the two coinitial sides of the parallelogram, then its diagonals are given by* $\overrightarrow{d_1}=\vec{a}+\vec{b}$ *and* $\overrightarrow{d_2}=\vec{b}-\vec{a}\implies\vec{a}=\frac{(\overrightarrow{d_1}+\overrightarrow{d_2})}{2}$ and $\vec{b}=\frac{(\overrightarrow{d_1}-\overrightarrow{d_2})}{2}$

Thus, area of the parallelogram = $|\vec{a}\times\vec{b}|=\left|\frac{(\overrightarrow{d_1}+\overrightarrow{d_2})}{2}\times\frac{(\overrightarrow{d_1}-\overrightarrow{d_2})}{2}\right|=\frac{1}{2}|\overrightarrow{d_1}\times\overrightarrow{d_2}|$

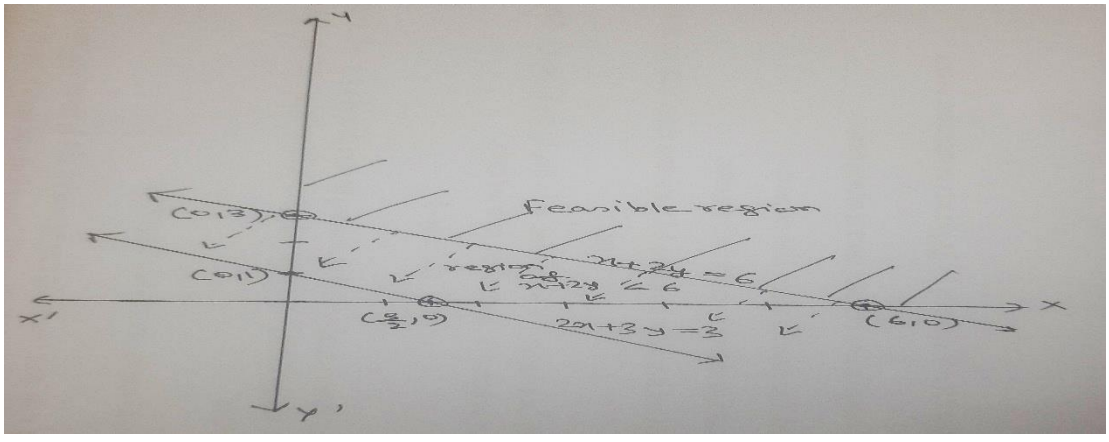
Required area = $\frac{1}{2}|\overrightarrow{d_1}\times\overrightarrow{d_2}|=\frac{1}{2}|(3\hat{\imath}+\hat{\jmath}-2\hat{k})\times(\hat{\imath}-2\hat{\jmath}+4\hat{k})|=\frac{7}{2}\sqrt{5}\ sq\ units.$ (2+ 1)

Q29/28) Consider the following Linear Programming Problem:

Minimise Z = x + 2y subject to $2x+3y\geq 3$, $x+2y\geq 6$, $x,y\geq 0$.

Show graphically that the minimum of Z occurs at more than two points

Ans)



The corner points of the **unbounded** feasible region are $A(6,0)$ and $B(0,3)$.

The values of Z at these corner points are as follows:

Corner point	Value Of Z = x + 2y
A(6, 0)	6
B(0, 3)	6

Since the graph of inequality x + 2y < 6 does not have any point common with the feasible area Minimum value of Z is 6 at both A (6, 0) and B (0, 3) and in fact at any point on the line segment AB.

(1.5 + 1.5)

Q30/27) Find the mean of the number of defective items in a sample of two items drawn one by one without replacement from an urn containing 6 items which includes 2 defective items. Assume that the items are identical in shape and size.

OR

A and B are two independent events such that P $(A \cap \bar{B}) = \frac{1}{4}$ *and* $P(\bar{A} \cap B) = \frac{1}{6}$. Find $P\ (A \cap B)$.

Ans) Let the random variable X denote the number of defective items drawn, then X = 0, 1, 2

X	0	1	2
P(X)	$\frac{4}{6}.\frac{3}{5} = \frac{12}{30} = \frac{2}{5}$	$2.\frac{4}{6}.\frac{2}{5} = \frac{16}{30} = \frac{8}{15}$	$\frac{2}{6}.\frac{1}{5} = \frac{2}{30} = \frac{1}{15}$
X.P(X)	0	$\frac{8}{15}$	$\frac{2}{15}$

Thus, E(X) = $\sum X.P(X) = 0 + \frac{8}{15} + \frac{2}{15} = \frac{10}{15} = \frac{2}{3}$ (2+1)

OR

$P(A)\ P(\bar{B}) = \frac{1}{4} \quad P(\bar{A})\ P(B) = \frac{1}{6}$

Let $P(A) = x \quad P(B) = y$

$$x(1-y) = \frac{1}{4}, \quad (1-x)y = \frac{1}{6} \Rightarrow x - y = \frac{1}{12}$$

eliminating y, we get $12x^2 - 13x + 3 = 0$

gives $x = \frac{1}{3}, \frac{3}{4}$

$P(A) = \frac{1}{3} \Rightarrow P(B) = \frac{1}{4}$

$P(A) = \frac{3}{4} \Rightarrow P(B) = \frac{2}{3}$

Thus, P (A∩B) = $P(A).P(B) = \frac{1}{12}$ *or* $\frac{1}{2}$ (1+ 1 + 1)

Q31) SET A1) A ladder 5cm long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 cm/s. How fast is the height on the wall decreasing when the foot of the ladder is 4cm away from the wall?

/Q26) SET A2) A particle is moving along the curve $6y = x^3 + 2$. Find the points on the curve at which the Y coordinate is changing 8 times as fast as the X coordinate.

Ans) SET A1) At any instant, let the ladder be leaning at a height of y cm along the wall and its foot be at a distance x cm from the wall. Then, we have $x^2 + y^2 = 25$ *or* $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ *or* $2x(2) + 2y\frac{dy}{dt} = 0\ i.e\ \frac{dy}{dt} = \frac{-2x}{y}$

Now, given x = 4cm, thus $\left.\frac{dy}{dt}\right|_{x=4} = \left.\frac{-2x}{y}\right|_{x=4} = \frac{-8}{3}$ cm/sec.

/ 26) SET A2) Given that, $\frac{dy}{dx} = 2$, *now as* $6y = x^3 + 2$ *so* $6\frac{dy}{dx} = 3x^2$ *or* $12 = 3x^2$ *or* $x = \pm 2$

So, the points on the curve are $(4, 11)$ *and* $\left(-4, \frac{-31}{3}\right)$ (2 + 1)

SECTION D

Q32/35) Find the value of p so that the lines $\frac{(1-x)}{3} = \frac{(7y-14)}{p} = \frac{(z-3)}{2}$ *and* $\frac{(7-7x)}{3p} = \frac{(y-5)}{1} = \frac{(6-z)}{5}$ are perpendicular to each other. Also, find the point of intersection of the lines $\frac{(x-1)}{2} = \frac{(y-2)}{3} = \frac{(z-3)}{4}$and $\frac{(x-4)}{5} = \frac{(y-1)}{2} = z$.

OR

Express the following lines in their vector form:

$l_1: x = 4\lambda + 5, y = -5\lambda + 7, z = -5\lambda - 3$ *and* $l_2: x = 7\mu + 8, y = \mu + 7, z = 3\mu + 5$

Also, find the shortest distance between these lines and obtain the acute angle between them.

Ans) The direction ratios of the given lines are $< -3, \frac{p}{7}, 2 >$ *and* $< -\frac{3p}{7}, 1, -5 >$

Thus, lines are perpendicular if $(-3).\left(-\frac{3p}{7}\right) + \frac{p}{7}.1 + 2.(-5) = 0$ *or* $p = 7$

Thus, the direction ratios of the $\frac{(1-x)}{3} = \frac{(7y-14)}{p} = \frac{(z-3)}{2}$ *or* $\frac{(x-1)}{-3} = \frac{(y-2)}{\frac{p}{7}} = \frac{(z-3)}{2}$ are $< -3, \frac{p}{7}, 2 >$ *or* $< -3, 1, 2 >$

Any point on the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is

$(2\lambda + 1, 3\lambda + 2, 4\lambda + 3),\ \lambda \in R$

For the point of intersection, these coordinates must satisfy $\frac{x-4}{5} = \frac{y-1}{2} = z$

$\Rightarrow \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = 4\lambda + 3$

$\Rightarrow \lambda = -1$

$\therefore$ point of intersection is $(-1, -1, -1)$ (2.5 + 2.5)

OR

We know that vector equation of a line is of the type $\vec{r} = \vec{a} + \lambda\vec{b}$

The vector form of the given lines is:

$l_1: \vec{r} = 5\hat{\imath} + 7\hat{\jmath} - 3\hat{k} + \lambda(4\hat{\imath} - 5\hat{\jmath} - 5\hat{k})$ *and* $l_2: \vec{r} = (8\hat{\imath} + 7\hat{\jmath} + 5\hat{k}) + \mu(7\hat{\imath} + \hat{\jmath} + 3\hat{k})$

Here, $\overrightarrow{a_1} = 5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$, $\overrightarrow{a_2} = (8\hat{\imath} + 7\hat{\jmath} + 5\hat{k}), \overrightarrow{b_1} = (4\hat{\imath} - 5\hat{\jmath} - 5\hat{k})$ *and* $\overrightarrow{b_2} = 7\hat{\imath} + \hat{\jmath} + 3\hat{k}$

So, the shortest distance between the two lines is given by,

$S.D = \left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})}{|(\overrightarrow{b_1} \times \overrightarrow{b_2})|}\right| = \left|\frac{(3\hat{\imath} + 0\hat{\jmath} + 8\hat{k}).(-10\hat{\imath} - 47\hat{\jmath} + 39\hat{k})}{\sqrt{3830}}\right| = \frac{282}{\sqrt{3830}}$ units. (3)

Acute angle between these lines is given by, $\cos\theta = \left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}\right| = \left|\frac{28 - 5 - 15}{\sqrt{66}\sqrt{59}}\right| = \frac{8}{\sqrt{66}\sqrt{59}}$ (2)

Q33/34) A toy rocket is fired, from a platform, vertically into the air, its height above the ground after t seconds is given by $s(t) = at^2 + bt + c, where\ a, b, c\ \in \boldsymbol{R}, a\ \neq 0\ and\ s(t)$ is measured in metres. After 10 seconds, the rocket is 16m above the ground; after 20 seconds, 22m and after 30 seconds, 25m. Write down a system of three linear equations in terms of a, b and c. Hence, find the values of a, b and c using matrix method.

Ans) We have $s(t) = at^2 + bt + c, and\ (10, 16), (20,22) and\ (30, 25) lie\ on\ the\ curve\ s(t)$

Thus, the system of linear equations is

100 a + 10 b + c = 16, 400 a + 20 b + c = 22 and 900 a + 30 b + c = 25

Now the given system of equations is expressible as

$$\begin{bmatrix}100 & 10 & 1\\ 400 & 20 & 1\\ 900 & 30 & 1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix} = \begin{bmatrix}16\\ 22\\ 25\end{bmatrix} \quad \text{Or} \quad AX = B, where\ X = \begin{bmatrix}a\\ b\\ c\end{bmatrix}\ and\ B = \begin{bmatrix}16\\ 22\\ 25\end{bmatrix}$$

Now $|A| = -2000 \neq 0 \Rightarrow A^{-1}\ exists$

The given system of equations is consistent with a unique solution given by

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix}a\\ b\\ c\end{bmatrix} = -\frac{1}{2000}\begin{bmatrix}-10 & 20 & -10\\ 500 & -800 & 300\\ -6000 & 6000 & -2000\end{bmatrix}\begin{bmatrix}16\\ 22\\ 25\end{bmatrix} = -\frac{1}{2000}\begin{bmatrix}30\\ -2100\\ -14000\end{bmatrix} = \begin{bmatrix}-\frac{3}{200}\\ \frac{21}{20}\\ 7\end{bmatrix}$$

$Thus, a = -\frac{3}{200}, b = \frac{21}{20} and\ c = 7$ is the required solution. (1.5 + 1.5 + 2)

Q34/33) Find $\frac{dy}{dx}\ where\ y = e^{xsin^2x} - (cosx)^x$

OR

$If\ \cos y = x\cos(a + y)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

Hence show that: $\sin a\frac{d^2y}{dx^2} + \sin 2(a + y)\frac{dy}{dx} = 0$

Ans) $Let\ y = u - v\ where\ u = e^{xsin^2x} and\ v = (cosx)^x. Thus, \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

Now, $u = e^{xsin^2x}\ or \frac{du}{dx} = e^{xsin^2x}(xsin2x + sin^2x)$

$v = (cosx)^x\ or\ log\ v = x\log(cosx), so\ \frac{dv}{dx} = (cosx)^x[\log(cosx) - x\ tanx]$

Thus, $\frac{dy}{dx} = e^{xsin^2x}(xsin2x + sin^2x) - (cosx)^x[\log(cosx) - x\ tanx]$ (2 + 2 + 1)

OR

$x = \frac{\cos y}{\cos(a+y)}\ Differentiating\ w.r.t\ y, on\ both\ sides, we\ get\ \frac{dx}{dy} = \frac{\sin[(a+y)-y]}{cos^2(a+y)}$

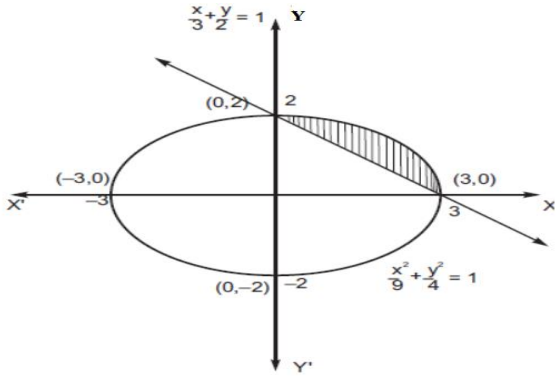
$Thus, \frac{dy}{dx} = \frac{cos^2(a + y)}{sina}$

$or\ sina.\frac{dy}{dx} = cos^2(a + y)\ Again\ differentiating\ both\ sides\ w.r.t.x, we\ get$

$sina\frac{d^2y}{dx^2} + \frac{dy}{dx}.0 = 2\cos(a + y)[-\sin(a + y)].\frac{dy}{dx} \Rightarrow sina\frac{d^2y}{dx^2} + sin2(a + y)\frac{dy}{dx} = 0$ (2.5 + 2.5)

Q35/32) Draw a rough sketch and using integration, find the area of the following region:
$\{(x, y): \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \leq \frac{x}{3} + \frac{y}{2}\}$.

Ans)



(Correct figure + shaded region + points of intersection) (1+0.5+0.5)

The required area = Shaded area =

$$= \int_0^3 (y_1 - y_2)\, dx$$
$$= \int_0^3 \left\{\frac{2}{3}\sqrt{9 - x^2} - \frac{2}{3}(3 - x)\right\} dx$$
$$= \frac{2}{3}\int_0^3 \left\{\sqrt{9 - x^2} - (3 - x)\right\} dx$$
$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} - 3x + \frac{x^2}{2}\right]_0^3$$
$$= \frac{2}{3}\left[\left(\frac{3}{2}\sqrt{0} + \frac{9}{2}\sin^{-1}(1) - 9 + \frac{9}{2}\right) - (0 + 0 - 0 + 0)\right]$$
$$= \frac{2}{3}\left[\frac{9}{2}\cdot\frac{\pi}{2} - \frac{9}{2}\right] = \frac{3}{2}(\pi - 2) \text{ sq. units.}$$

(3)

SECTION E

This section comprises 3 case study/passage-based questions of 4 marks each. First two case study questions have 3 subparts (i), (ii), & (iii) of 1, 1 & 2 marks respectively. The third case study question has two sub parts (i) & (ii) of 2 marks each respectively.

Case Study – 1

Q36) Pratibha Vikas is an innovative program by the Government of Delhi, where cultural and literacy competitions are held between schools at cluster, block, district and state levels. One of those competitions - Yogasana, is conducted under two categories: Middle school and High school. From South Delhi district, three students from middle school and two students from high school were selected for the state level.

Let $M = \{m_1, m_2, m_3\}$ and $H = \{h_1, h_2\}$, represent the set of students from middle school and high school respectively who got selected for the state level from that district.

A relation $R: M \to M$ is defined by $R = \{(x, y)$: x and y are students from the same category$\}$.

On the basis of the above information, answer the following questions.

(i) Check if the relation R is reflexive. Justify your answer.

(ii) Check if the relation R is symmetric. Justify your answer.

(iii) (a) Check if the relation R is transitive. Is R an equivalence relation? Justify your answer.

OR

(iii) (b) Let a function $f: M \to H$ is defined as $f = \{(m_1, h_1), (m_2, h_2), (m_3, h_2)\}$. Check whether the function f is one-one and onto. Justify your answer.

Ans) (i) Yes R is refelxive as $(m_1, m_1), (m_2, m_2) and \left(m_3, m_3\right) \in R$.

(ii) Yes, R is symmetric because if $(m_1, m_2) \in R$ *or* m_1 and m_2 are students from the same category, $\Longrightarrow (m_2, m_1) \in R$.

(iii) (a) Yes, R is transitive as if (x, y) $\in R$ *and* $(y, z) \in R$

$\Longrightarrow x, y$ *and* z *are students of the same category or* $(x, z) \in R$.

Since, R is rerfelexive, synmmetric and tyransitive as well, it is an equivalence relation.

OR

(iii) (b) $f: M \to H$ is defined as $f = \{(m_1, h_1), (m_2, h_2), (m_3, h_2)\}$, is not 1-1 but onto as m_2 *and* m_3 have the same image h_2, *however Range* f = *Codomain* $f = H$. (1+1 + 2)

Case Study – 2

Q37) A student Shivam is running on a playground along the curve given by $y = x^2 + 7$. Another student Manita standing at point (3, 7) on playground wants to hit Shivam by paper ball when Shivam is nearest to Manita.

Based on above information, answer the following questions:

(i) Let at any instant while running along the curve $y = x^2 + 7$., Shivam's position be (x, y). Find the expression for the distance (D) between Shivam and Manita in terms of ' x'.

(ii) Find the critical point(s) of the distance function.

(iii) (a) What is the distance between Shivam and Manita when they are at least distance from each other.

OR

(iii) (b)Find the position of Shivam, when he is closest to Manita.

Ans) (i) For all values of x, $y = x^2 + 7$

$\therefore$ Shivam's position at any point of x will be $\left(x, x^2 + 7\right)$

The measure of the distance between Shivam and Manita, i.e., D

$$D = \sqrt{(x-3)^2 + \left(x^2 + 7 - 7\right)^2} = \sqrt{(x-3)^2 + x^4}$$

(ii)

$$D = \sqrt{(x-3)^2 + x^4}$$

$$\text{Let } \Delta = D^2 = (x-3)^2 + x^4$$

Now,

$$\frac{d}{dx}(\Delta) = 2(x-3) + 4x^3 = 4x^3 + 2x - 6$$

$$\frac{d}{dx}(\Delta) = 0 \Rightarrow x = 1$$

(iii) (a)

$\Delta''(x) = 12x^2 + 2$

Clearly, $\Delta''(x) = 12x^2 + 2 > 0$ at $x = 1$

$\therefore$ Value of x for which D will be minimum is 1.

For $x = 1, y = 8$.

Therefore, required distance $= D = \sqrt{(1-3)^2 + (1)^4} = \sqrt{4+1} = \sqrt{5}$

$\Delta''(x) = 8x^2 + 2$

Clearly, $\Delta''(x) = 8x^2 + 2 > 0$ at $x = 1$

$\therefore$ Value of x for which D will be minimum is 1.

For $x = 1, y = 8$.

Therefore, required distance $= D = \sqrt{(1-3)^2 + (1)^4} = \sqrt{4+1} = \sqrt{5}$

OR

(iii)(b)

$\Delta''(x) = 12x^2 + 2$

Clearly, $\Delta''(x) = 12x^2 + 2 > 0$ at $x = 1$

$\therefore$ Value of x for which D will be minimum is 1.

For $x = 1, y = 8$.

Therefore, required distance $= D = \sqrt{(1-3)^2 + (1)^4} = \sqrt{4+1} = \sqrt{5}$

Thus, the required position for Shivam is $(1, 8)$ when he is closest to Manita. (1+1+2)

Case Study – 3

Q38) Shyam and Radha wanted to play a game with balls of two different colors - red and blue. Shyam has two boxes I and II. Box I contains 3 red and 6 blue balls. Box II contains 5 red and 'n' blue balls. One of the two boxes is selected at random by Radha, and then she draws a ball from that box at random.

Based on the given information, answer the following questions.

(i) Shyam notices that probability that given the ball drawn by Radha is red, it being taken out from the bag II is $\frac{3}{5}$.Then Radha asks him about the value of n. What is the value of 'n'?

(ii) What is the probability that the ball drawn is found to be red?

Ans) (i) Given that $P(B2/R) = \frac{3}{5}$ or $\frac{3}{5} = \frac{P(R/B2).P(B2)}{P(R/B2).P(B2) + P(R/B1).P(B1)} = \frac{\frac{1}{2}.\frac{5}{(5+n)}}{\frac{1}{2}.\frac{5}{(5+n)} + \frac{1}{2}.\frac{3}{9}}$

or $\frac{3}{5} = \frac{\frac{5}{(5+n)}}{\frac{5}{(5+n)} + \frac{1}{3}}$ or $n = 5$

(ii)Required, P(R) = $P(R/B2).P(B2) + P(R/B1).P(B1) = \frac{5}{12}$ (2+2)