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CLASS XII
POST MID-TERM EXAM (2024 – 25)
MATHEMATICS (041)
Set A1

Time :3hrs. **M.M.80**

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 case-based assessment (4 marks) with sub parts.
7. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

SECTION A

(Multiple Choice Questions) Each question carries 1 mark.

Q1) X and Y are two matrices such that the transpose of (X + Y) is $\begin{bmatrix}2 & -1\\7 & 6\end{bmatrix}$.
$If\ Y = \begin{bmatrix}3 & -2\\6 & 0\end{bmatrix}$, then which of the following is correct?
(a) X = $\begin{bmatrix}-1 & 1\\1 & 6\end{bmatrix}$ (b) X = $\begin{bmatrix}-1 & 9\\-7 & 6\end{bmatrix}$ (c) X = $\begin{bmatrix}1 & -9\\7 & -6\end{bmatrix}$ (d) X = $\begin{bmatrix}-4 & 4\\0 & 7\end{bmatrix}$

Q2) If $\begin{bmatrix}-5 & 6 & 7\\6 & 2 & m^2-6\\7 & 3 & 0\end{bmatrix}$ is a symmetric matrix, then $m =$
(a) 0 (b) -2 (c) 2 (d) ± 3

Q3) If for a square matrix B, $B.(adjB) = \begin{bmatrix}-25 & 0 & 0\\0 & -25 & 0\\0 & 0 & -25\end{bmatrix}, then\ |B| + |adj\ B| =$
(a)$25^2 \times 24$ (b) (-25) + 1 (c) 25×24 (d) $(-25)^2 + 25$

Q4) For the matrix $A = \begin{bmatrix}1 & -1 & 2\\3 & 0 & -2\\1 & 0 & 3\end{bmatrix}$, the value of $|A^{-1}|$ will be
$(a)\ \frac{1}{11}$ (b)$-\frac{1}{11}$ (c) -11 (d) 11

Q5) The domain of the function $cos^{-1}(2x-3)\ is$
(a) $[-1,1]$ (b) $[0,\pi]$ (c) [1,2] (d) $(-1,1)$

Q6) The rate of change of the surface area of the sphere of radius r when the radius is increasing at the rate of 2 cm/s is proportional to
(a)$\frac{1}{r^2}$ (b) $\frac{1}{r}$ (c) r (d) r^2

Q7) If for a square matrix A, $A^2 - 3A + I = 0$ & $A^{-1} = xA + y\,I, then\ \ value\ of\ x + y\ is$

(a) -2 (b) 2 (c) 3 (d) -3

Q8) $\int \frac{dx}{(1-x)\sqrt{x}} =$

(a) $\frac{1}{2} log \left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right| + c$ (b) $-\frac{1}{2} log \left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right| + c$ (c) $log \left|\frac{1-\sqrt{x}}{1+\sqrt{x}}\right| + c$ (d) $log \left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right| + c$

Q9) If $\int e^{-log\,x} dx = f(x) + k, then f(x)$ is

(a) $\frac{x^3}{3}$ (b) $\log|x|$ (c) $-\frac{2}{x}$ (d) $\frac{1}{x}$

Q10) Area bounded by the parabola $y^2 = 4ax$ and its latus-rectum, is

(a) $8a^2$ (b) $\frac{4a^2}{3}$ (c) $\frac{8a^2}{3}$ (d) $\frac{3a^2}{4}$

Q11) The corner points of the feasible region determined by the system of linear constraints are (2, 4), (6, 7), (0, 8). Let Z = 3x – 5y be the objective function. The difference between maximum and minimum value of Z is

(a)26 (b) 14 (c) 17 (d) 23

Q12) The solution of $\frac{dy}{dx} - \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is

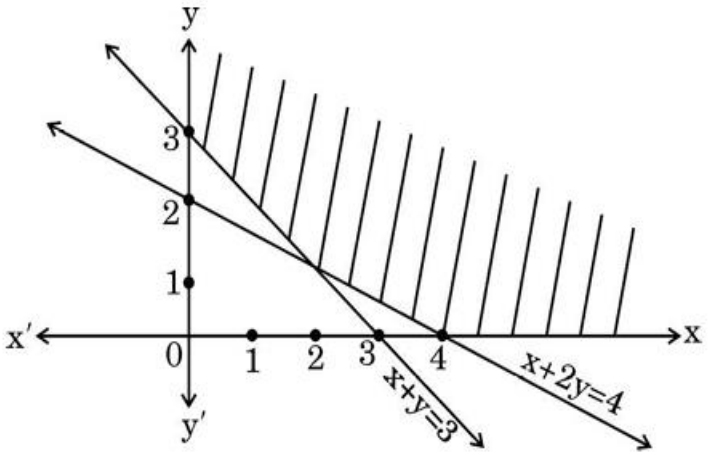
(a) $sin^{-1} x + y = c$ (b) $sin^{-1} x - sin^{-1} y = c$

(c) $sin^{-1} x + \sin^{-1} y = c$ (d) $tan^{-1} x + cot^{-1} y = c$

Q13) If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ is a non-zero scalar, then $\lambda\vec{a}$ is unit vector if

(a) $\lambda = 1$ (b) $\lambda = -1$ (c) $a = |\lambda|$ (d) $a = \frac{1}{|\lambda|}$

Q14) The feasible region of a linear programming problem is shown in the figure below:



Which of the following are the possible constraints?

(a) $x + 2y \geq 4, x + y \leq 3, x, y \geq 0$ (b) $x + 2y \leq 4, x + y \leq 3, x, y \geq 0$

(c) $x + 2y \geq 4, x + y \geq 3, x, y \geq 0$ (d) $x + 2y \geq 4, x + y \leq 3, x, y \leq 0$

Q15) If m and n are the order and degree, respectively of the differential equation

$7x\left(\frac{dy}{dx}\right)^4 - \frac{d^3y}{dx^3} - 6y = log\, x$, then the value of m + n is

(a) 1 (b) 2 (c) 3 (d) 4

Q16) If $(\hat{\imath} + 3\hat{\jmath} + 8\hat{k}) \times (3\hat{\imath} - \lambda\hat{\jmath} + \mu\hat{k}) = \vec{0}$, then the value of λ is

(a) 27 (b) 9 (c) –9 (d) – 1

Q17) Value of the constant 'k' so that the function $f(x) = \begin{cases} \frac{k|x|}{x}, if x < 0 \\ 3, if x \geq 0 \end{cases}$ is continuous at $x = 0$, is

(a) -3 (b) 3 (c) $-\frac{1}{\sqrt{2}}$ (d) $-\frac{\pi}{4}$

Q18) If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is equal to

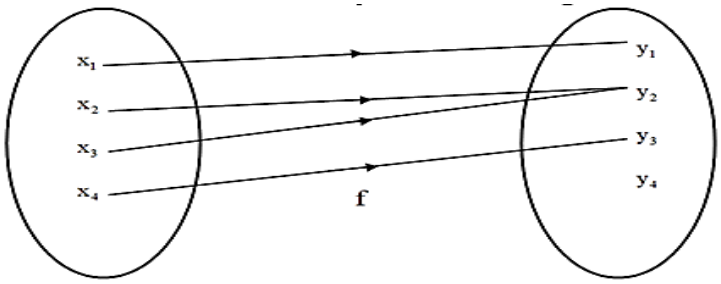
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) 1

ASSERTION-REASON BASED QUESTION:

In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19) **Assertion(A):** A function f shown below by the arrow diagram, is one-one.



Reason (R): A function $f: A \to B$ is one-one if $f(\alpha) = f(\beta)$ implies $\alpha = \beta$ for all $\alpha, \beta \in A$

Q20) **Assertion(A):** The function $f(x) = \begin{cases} 5 - 2x^2, & if\ x > 1 \\ x^2 + 1, & if\ x \leq 1 \end{cases}$ is not differentiable at x = 1.

Reason (R): A real function continuous at x = c is always differentiable as well at x = c.

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

Q21) Consider the statement "There exists at least one value of $b \in \boldsymbol{R}$ *for which* $f(x) = \frac{b}{x}$, $b \neq 0$ *is strictly increasing in* $\boldsymbol{R} - \{0\}$. State True or False. Justify your answer.

Q22) If the projection of the vector $\lambda\,\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on the vector $2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4, then find the value of λ.

Q23) Find $\frac{dx}{dy}$ if $x = a\,cos^2\,t\;and\;y = b\,sin^3\,t$

OR

If $y = \sqrt{\cos x + y}$, then prove that $\frac{dy}{dx} = \frac{\sin x}{1-2y}$

Q24) Find the value of k if $\sin^{-1}(k\,tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$.

Q25) Find a vector $\vec{r}$ of magnitude $5\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}\;and\;\frac{\pi}{2}$ with the y-axis and z-axis, respectively and an acute angle $\alpha\;with\;X\;axis$.

OR

A person standing at O (0, 0, 0) is watching an aeroplane which is at the coordinate point A (4, 0, 3). At the same time, he saw a bird at the coordinate point B (0, 0, 1). Find the angles which $\overrightarrow{BA}$ makes with X, Y and Z axes.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each.)

Q26) Evaluate: $\int_0^{\pi/2} [\log sinx - \log(2\cos x)]\,dx$

OR

Evaluate $\int \frac{(3sinx-2)cosx}{(4+\,sin^2x-4sinx)}\,dx$

Q27) The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6, 0 \le x \le 12$, "m" being a constant, where f(x) is the temperature in ^{0}F at x days. If 6 is a critical point of the function, then find the value of the constant "m". Also, find the points of absolute maximum and absolute minimum in the interval [0, 12] along with the absolute maximum and minimum values.

Q28) Find the vector equation of the line passing through the point (1, 3, – 7) and perpendicular to the two lines

$$\frac{x-6}{3} = \frac{y+23}{-16} = \frac{z-0}{7}\;and\;\frac{x-5}{3} = \frac{y+9}{8} = \frac{z+5}{-5}$$

OR

Using vectors, derive the expression for evaluating the area of a parallelogram in terms of its diagonal vectors. Hence, find the area of a parallelogram whose diagonals are given by $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$.

Q29) Consider the following Linear Programming Problem:
Minimise Z = x + 2y subject to $2x + 3y \ge 3$, $x + 2y \ge 6$, $x, y \ge 0$.
Show graphically that the minimum of Z occurs at more than two points

Q30) Find the mean of the number of defective items in a sample of two items drawn one by one without replacement from an urn containing 6 items which includes 2 defective items. Assume that the items are identical in shape and size.

OR

A and B are two independent events such that P (A $\cap \bar{B}$) $= \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find $P\ (A \cap B)$.

Q31) A ladder 5cm long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 cm/s. How fast is the height on the wall decreasing when the foot of the ladder is 4cm away from the wall?

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each).

Q32) Find the value of p so that the lines $\frac{(1-x)}{3} = \frac{(7y-14)}{p} = \frac{(z-3)}{2}$ and $\frac{(7-7x)}{3p} = \frac{(y-5)}{1} = \frac{(6-z)}{5}$ are perpendicular to each other. Also, find the point of intersection of the lines $\frac{(x-1)}{2} = \frac{(y-2)}{3} = \frac{(z-3)}{4}$and $\frac{(x-4)}{5} = \frac{(y-1)}{2} = z$.

OR

Express the following lines in their vector form:

$l_1: x = 4\lambda + 5, y = -5\lambda + 7, z = -5\lambda - 3$ and $l_2: x = 7\mu + 8, y = \mu + 7, z = 3\mu + 5$

Also, find the shortest distance between these lines and obtain the acute angle between them.

Q33) A toy rocket is fired, from a platform, vertically into the air, its height above the ground after t seconds is given by $s(t) = at^2 + bt + c, where\ a, b, c \in \boldsymbol{R}, a \neq 0\ and\ s(t)$ is measured in metres. After 10 seconds, the rocket is 16m above the ground; after 20 seconds, 22m and after 30 seconds, 25m. Write down a system of three linear equations in terms of a, b and c. Hence, find the values of a, b and c using matrix method.

Q34) Find $\frac{dy}{dx}$ $where\ y = e^{xsin^2x} - (cosx)^x$

OR

$If\ \cos y = x\cos(a + y)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

Hence show that: $\sin a \frac{d^2y}{dx^2} + \sin 2(a + y)\frac{dy}{dx} = 0$

Q35) Draw a rough sketch and using integration, find the area of the following region:

$\{(x, y): \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \leq \frac{x}{3} + \frac{y}{2}\}$.

SECTION E

This section comprises 3 case study/passage-based questions of 4 marks each. First two case study questions have 3 subparts (i), (ii), & (iii) of 1, 1 & 2 marks respectively. The third case study question has two sub parts (i) & (ii) of 2 marks each respectively.

Case Study – 1

Q36) Pratibha Vikas is an innovative program by the Government of Delhi, where cultural and literacy competitions are held between schools at cluster, block, district and state levels. One of those

competitions - Yogasana, is conducted under two categories: Middle school and High school. From South Delhi district, three students from middle school and two students from high school were selected for the state level.

Let $M = \{m_1, m_2, m_2\}$ and $H = \{h_1, h_2\}$, represent the set of students from middle school and high school respectively who got selected for the state level from that district.

A relation $R: M \to M$ is defined by $R = \{(x, y)$: x and y are students from the same category$\}$.
On the basis of the above information, answer the following questions.

(i) Check if the relation R is reflexive. Justify your answer.

(ii) Check if the relation R is symmetric. Justify your answer.

(iii) (a) Check if the relation R is transitive. Is R an equivalence relation? Justify your answer.

OR

(iii) (b) Let a function $f: M \to H$ is defined as $f = \{(m_1, h_1), (m_2, h_2), (m_3, h_2)\}$. Check whether the function f is one-one and onto. Justify your answer.

Case Study – 2

Q37) A student Shivam is running on a playground along the curve given by $y = x^2 + 7$. Another student Manita standing at point (3, 7) on playground wants to hit Shivam by paper ball when Shivam is nearest to Manita.
Based on above information, answer the following questions:

(i) Let at any instant while running along the curve $y = x^2 + 7$., Shivam's position be (x, y). Find the expression for the distance (D) between Shivam and Manita in terms of ' x'.
(ii) Find the critical point(s) of the distance function.
(iii) (a) What is the distance between Shivam and Manita when they are at least distance from each other.

OR

(iii) (b)Find the position of Shivam, when he is closest to Manita.

Case Study – 3

Q38) Shyam and Radha wanted to play a game with balls of two different colors - red and blue. Shyam has two boxes I and II. Box I contains 3 red and 6 blue balls. Box II contains 5 red and 'n' blue balls. One of the two boxes is selected at random by Radha, and then she draws a ball from that box at random.
Based on the given information, answer the following questions.

(i) Shyam notices that probability that given the ball drawn by Radha is red, it being taken out from the bag II is $\frac{3}{5}$.Then Radha asks him about the value of n. What is the value of 'n'?

(ii) What is the probability that the ball drawn is found to be red?